Foundation for Inference

IS381 - Statistics and Probability with R

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October 13, 2025

One Minute Paper Results

What was the most important thing you learned during this class?

What important question remains unanswered for you?

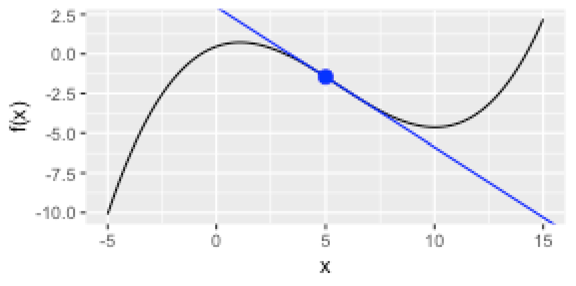
Crash Course in Calculus

Crash Course in Calculus

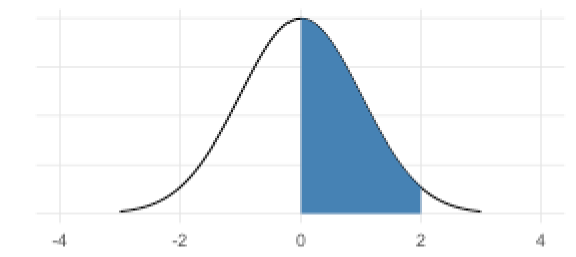
There are three major concepts in calculus that will be helpful to understand:

Limits - the value that a function (or sequence) approaches as the input (or index) approaches some value.

Derivatives - the slope of the line tangent at any given point on a function.

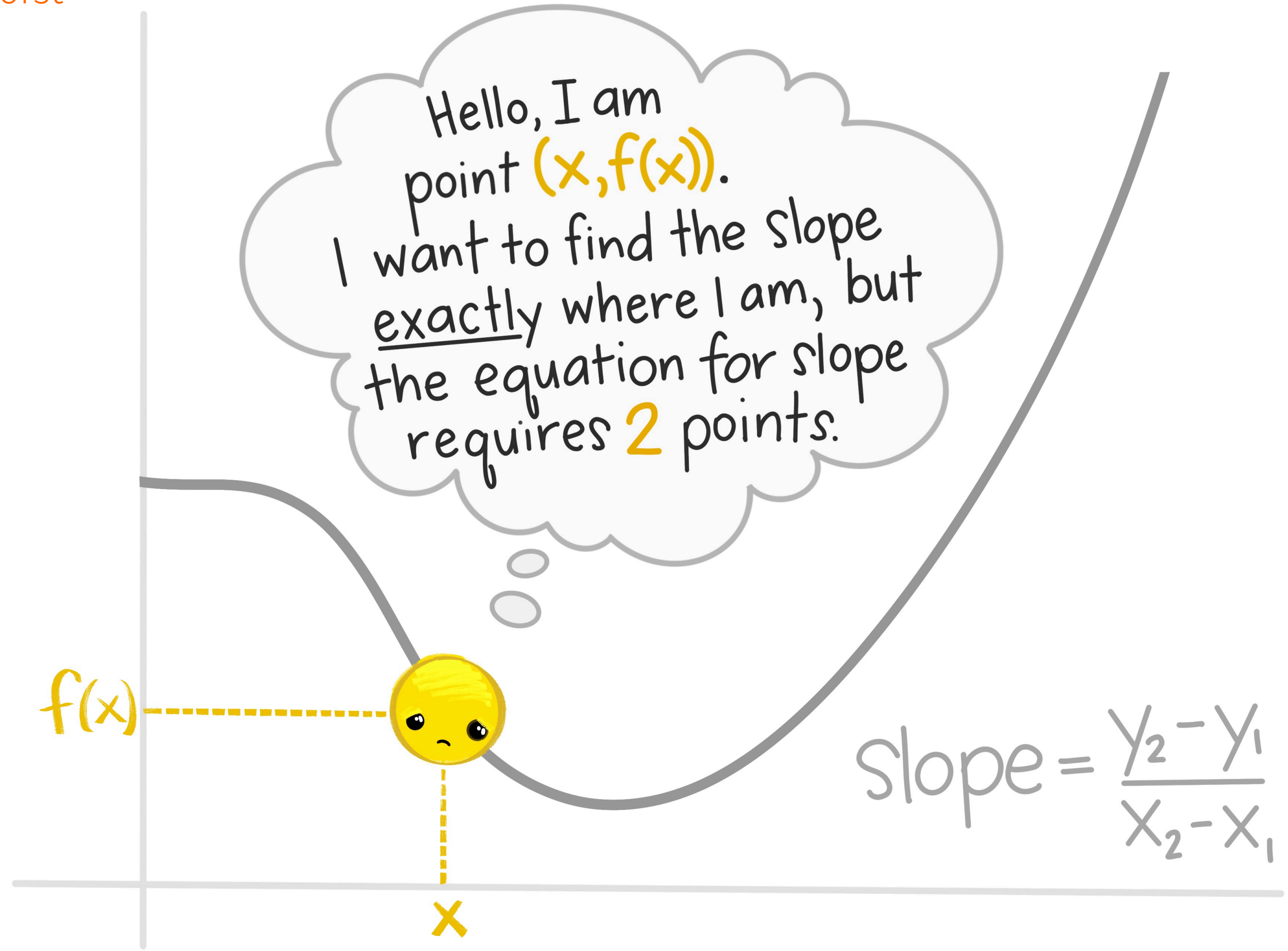


Integrals - the area under the curve.



Derivatives

Source: @allison_horst



Derivatives

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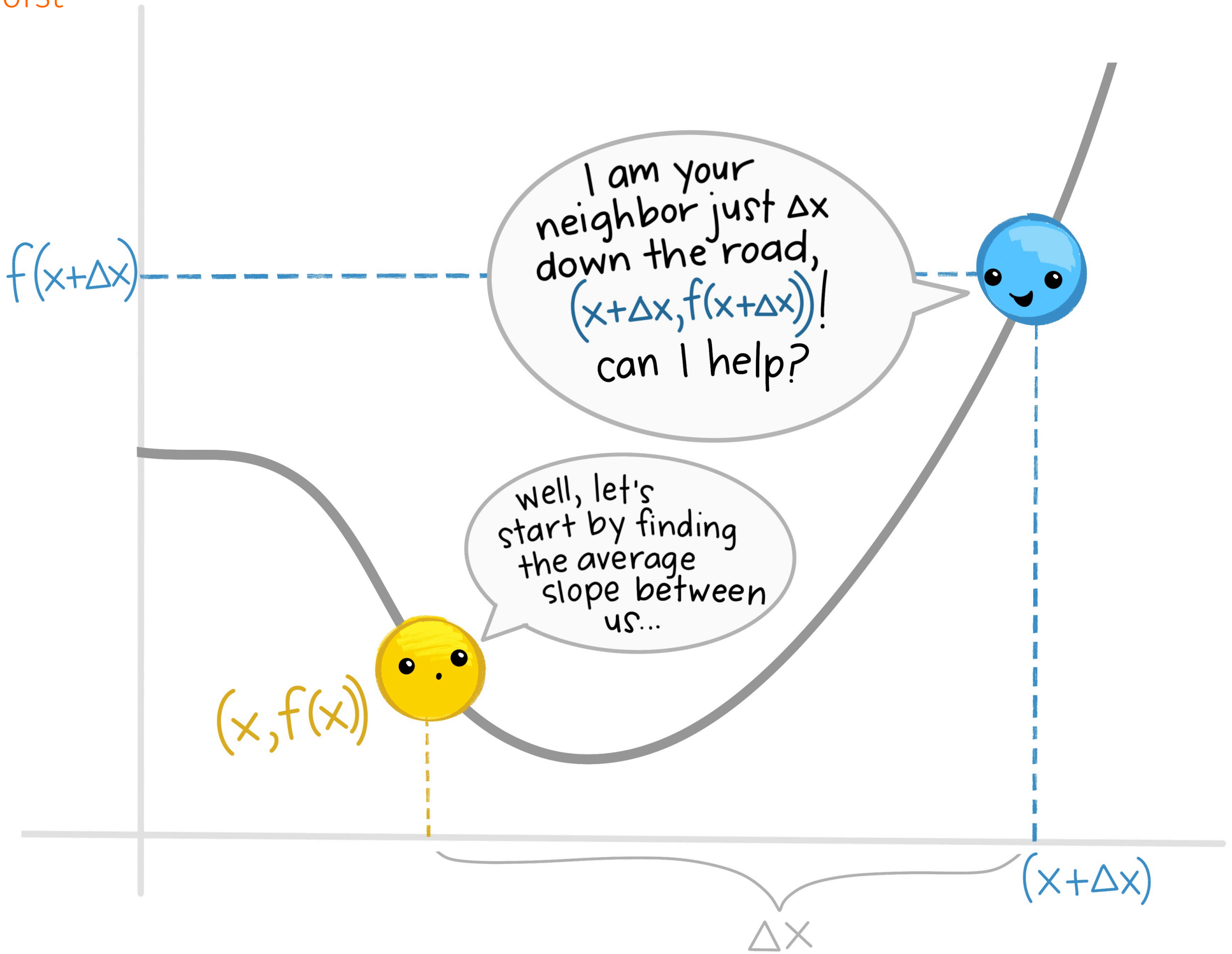

Derivatives

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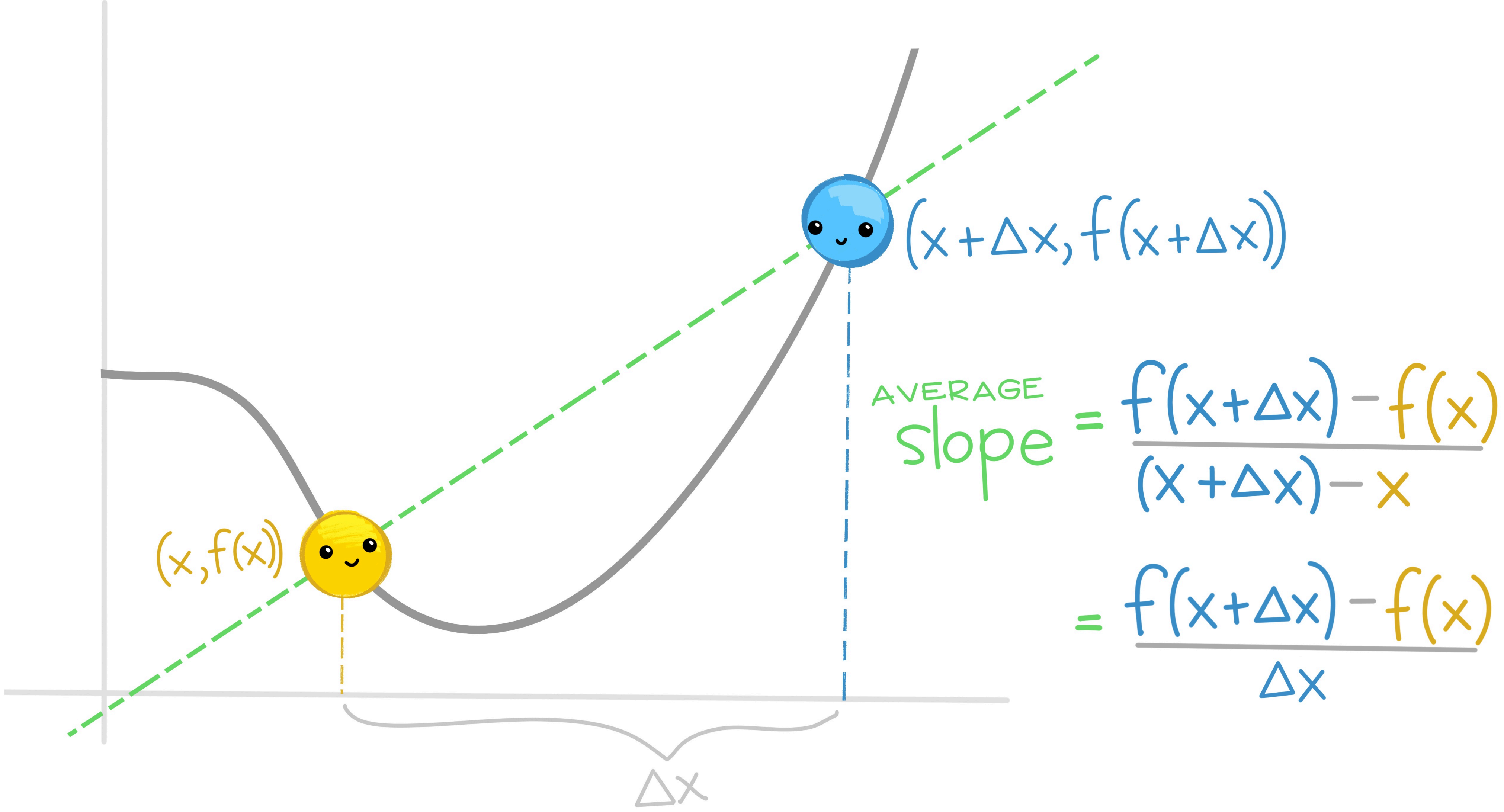

Derivatives

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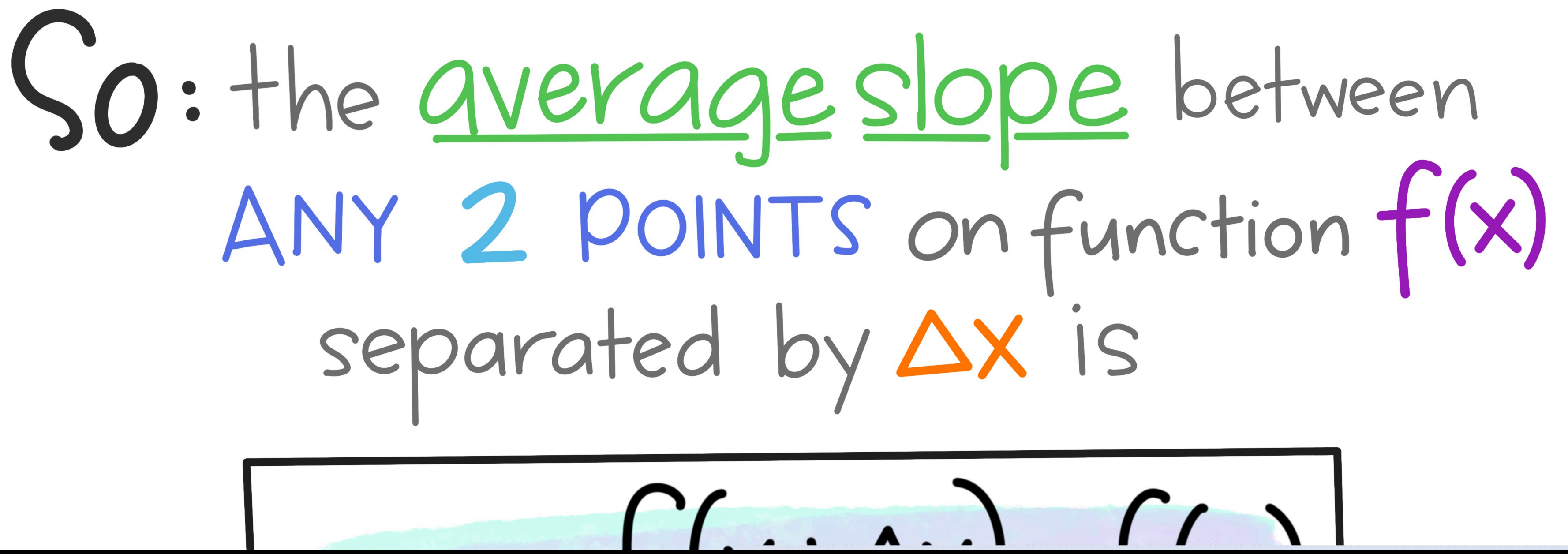

Derivatives

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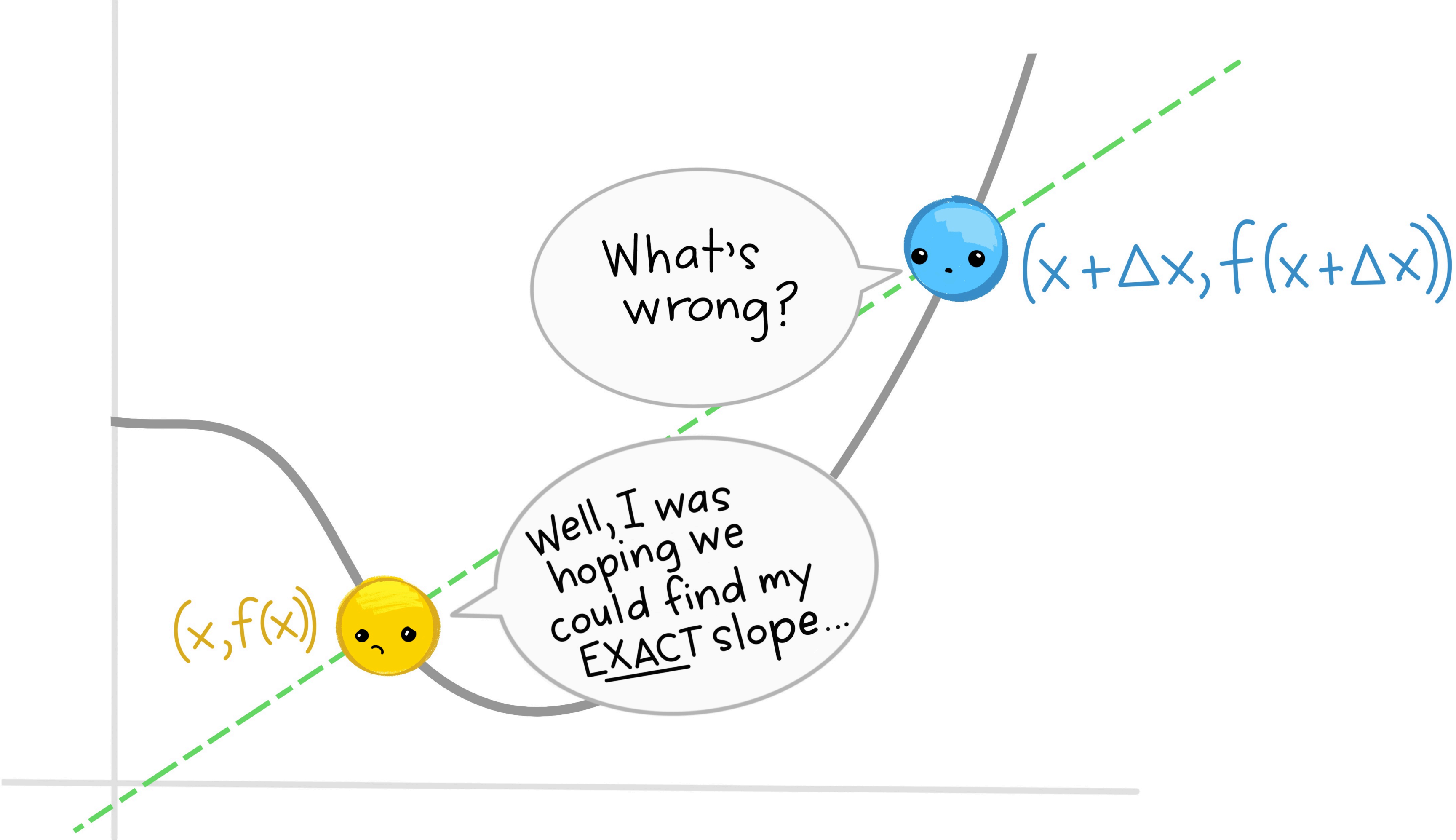

Derivatives

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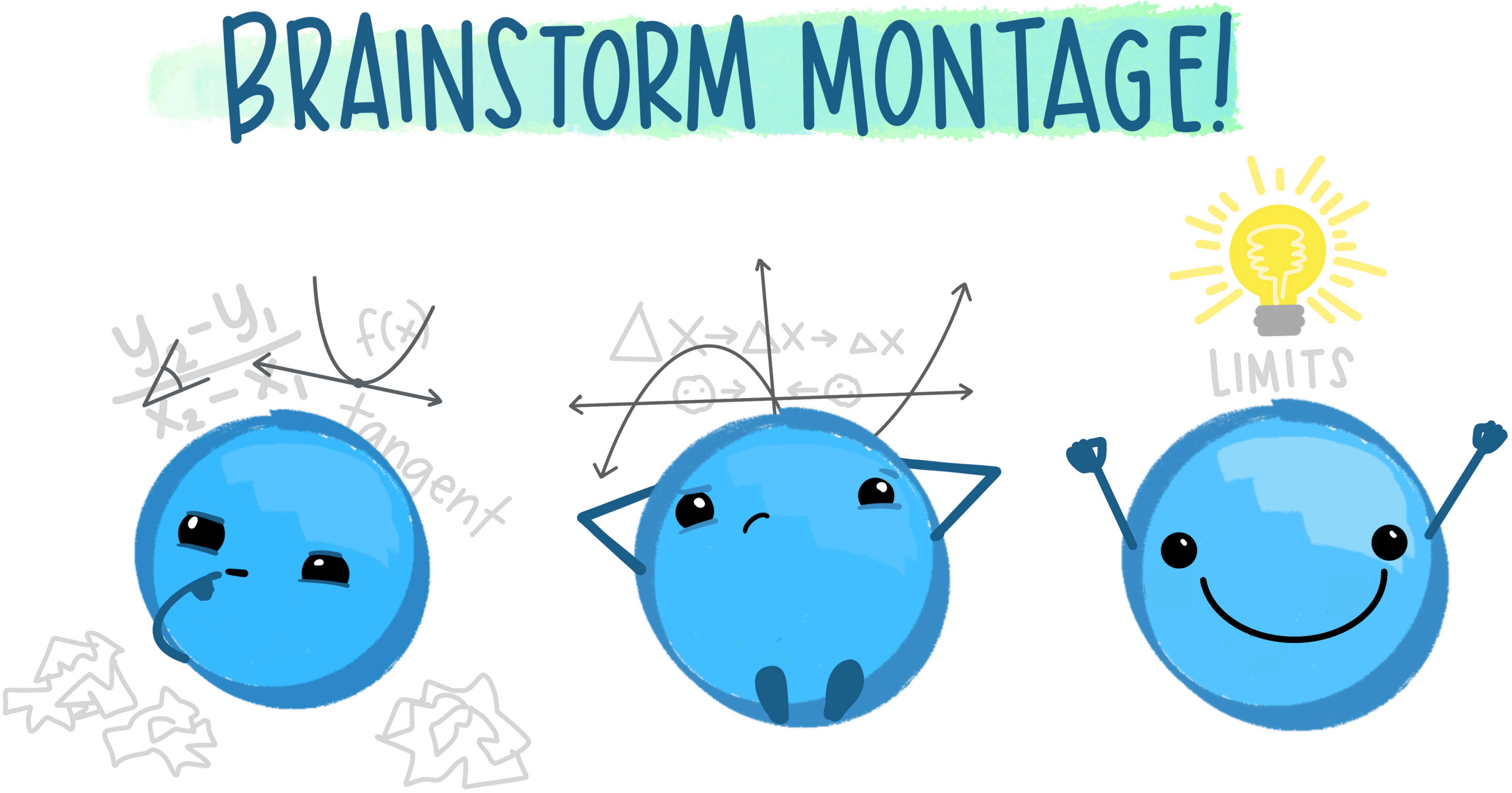

Derivatives

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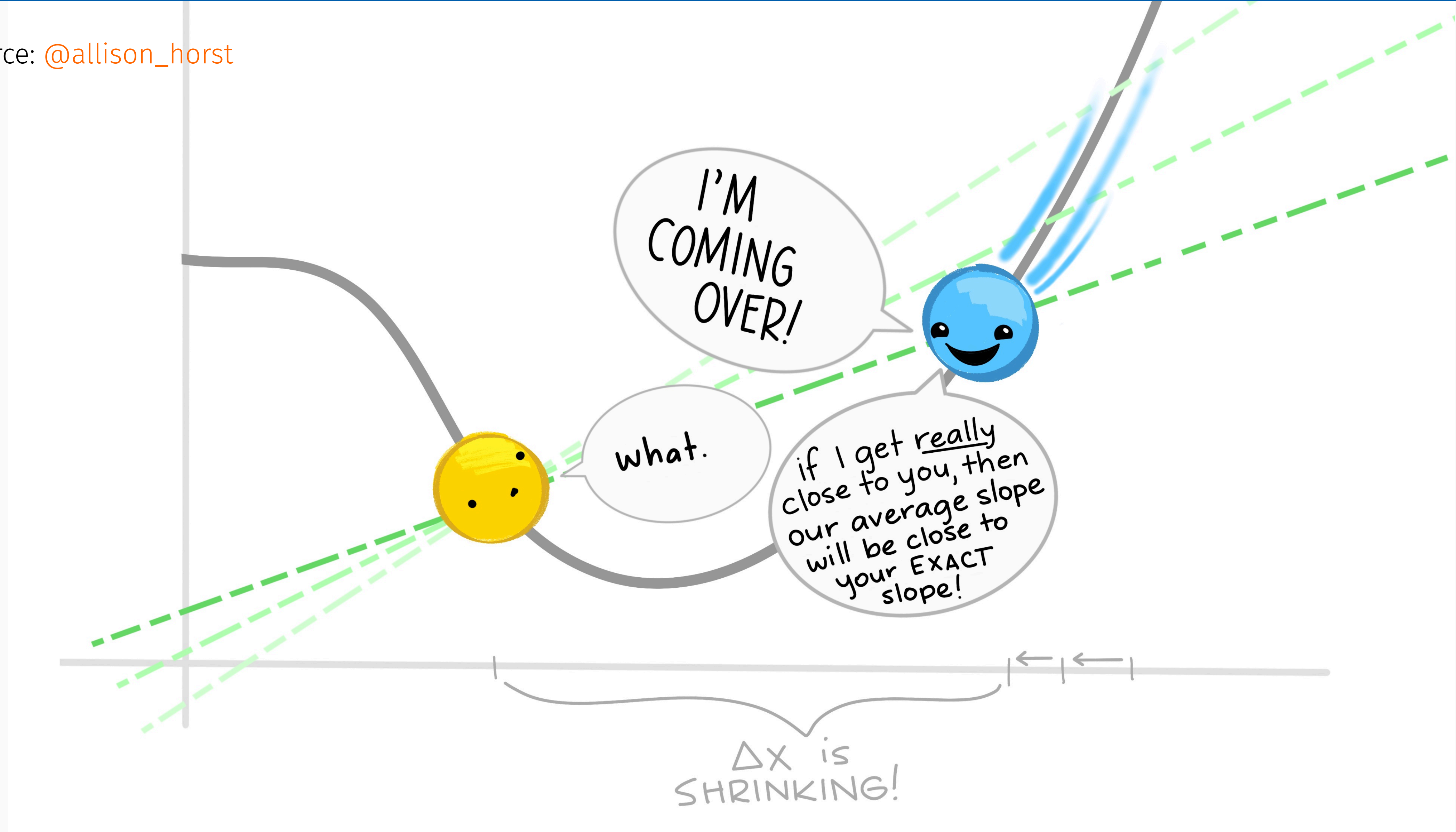

Derivatives

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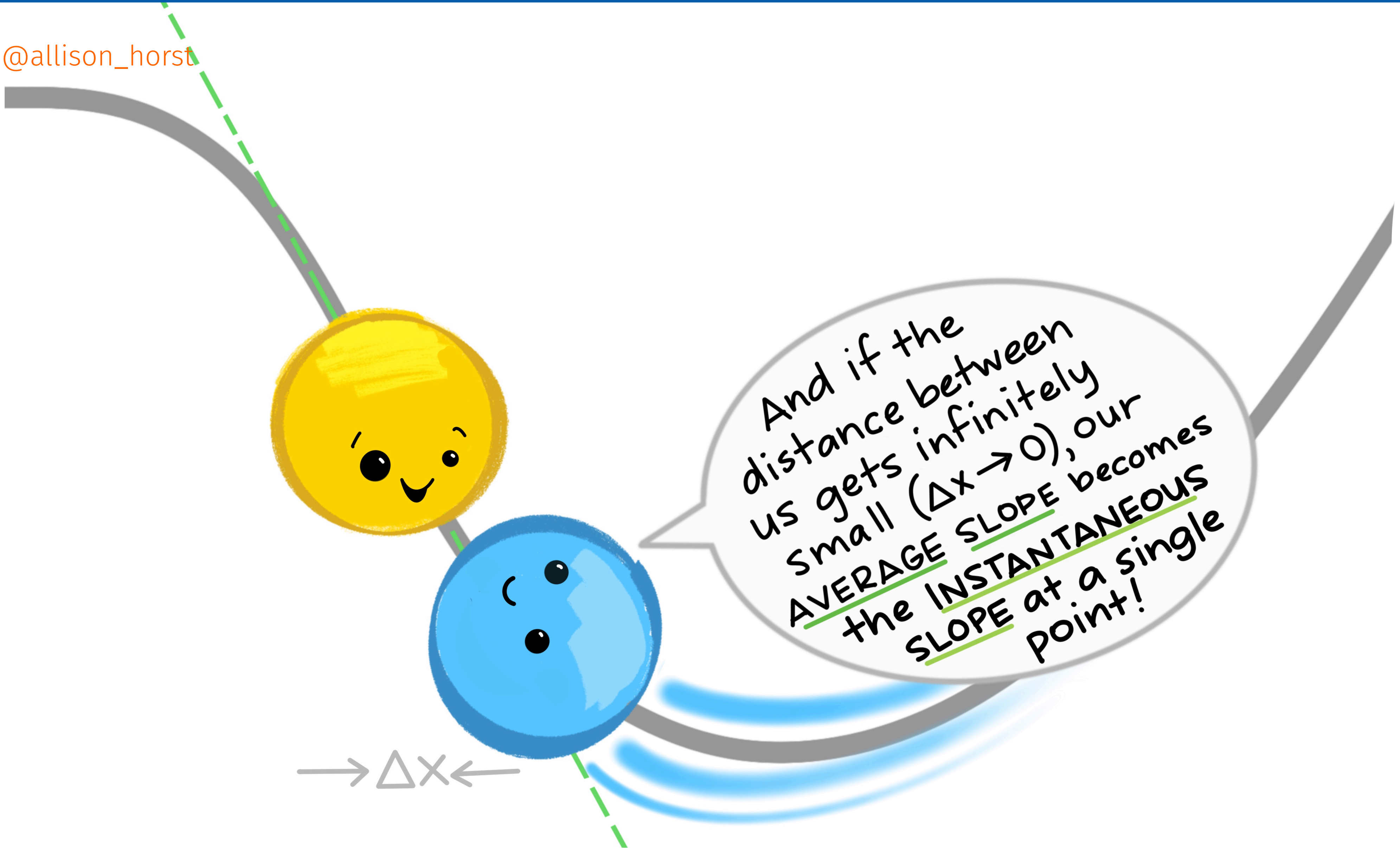

Derivatives

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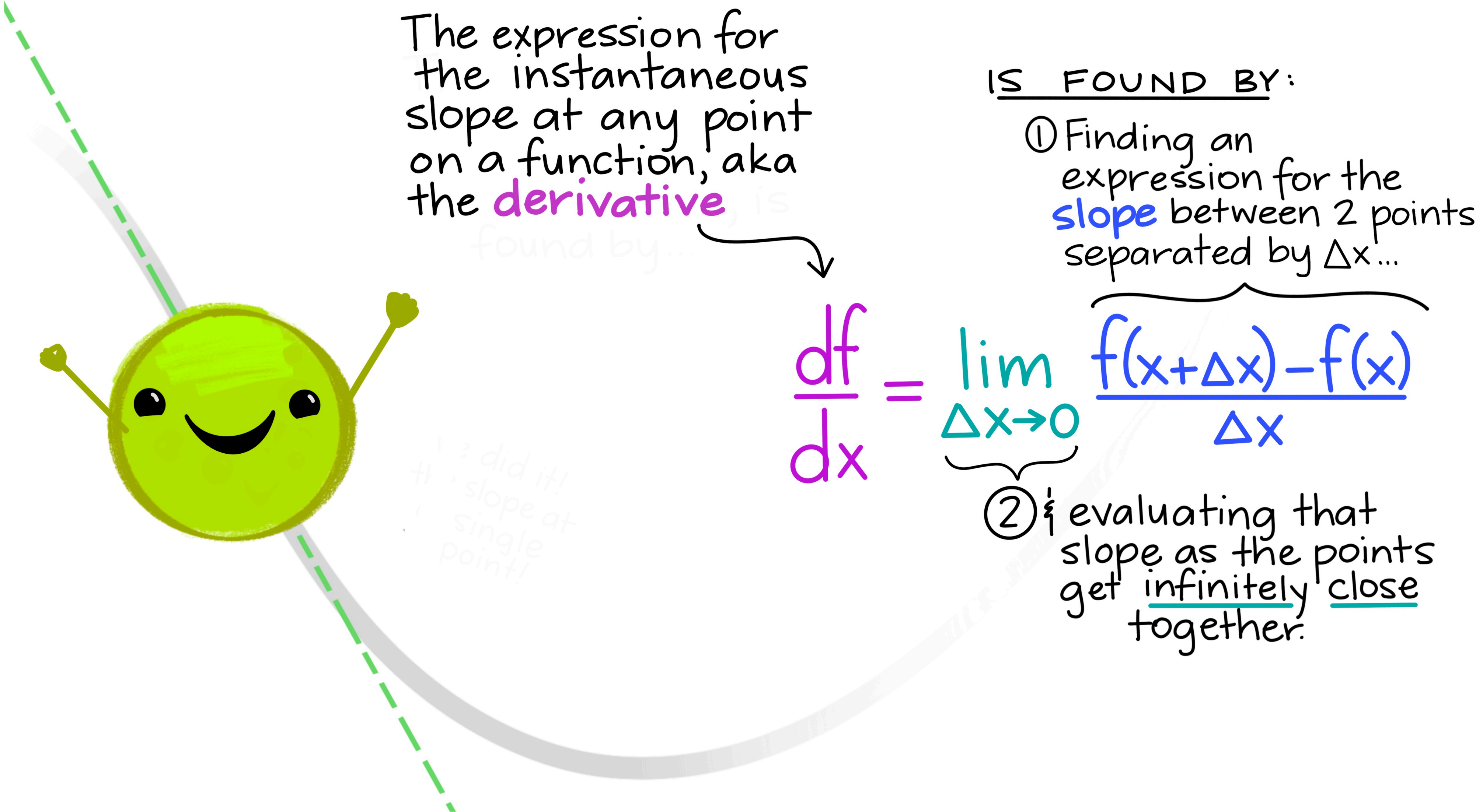
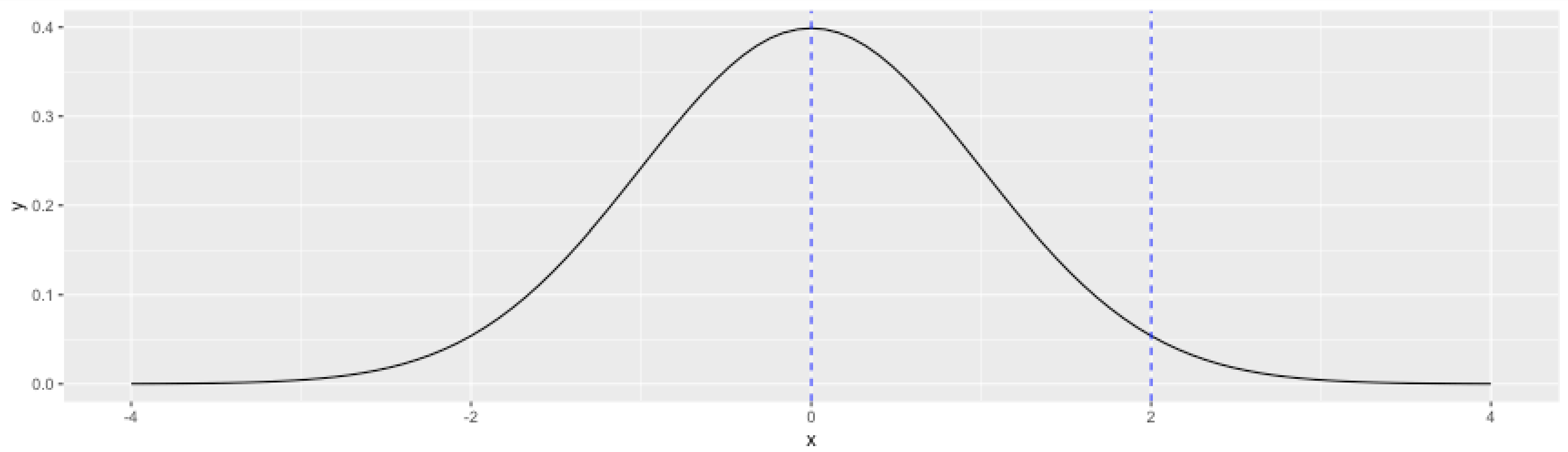


Function for Normal Distribution

$$f\left(x|\mu,\sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

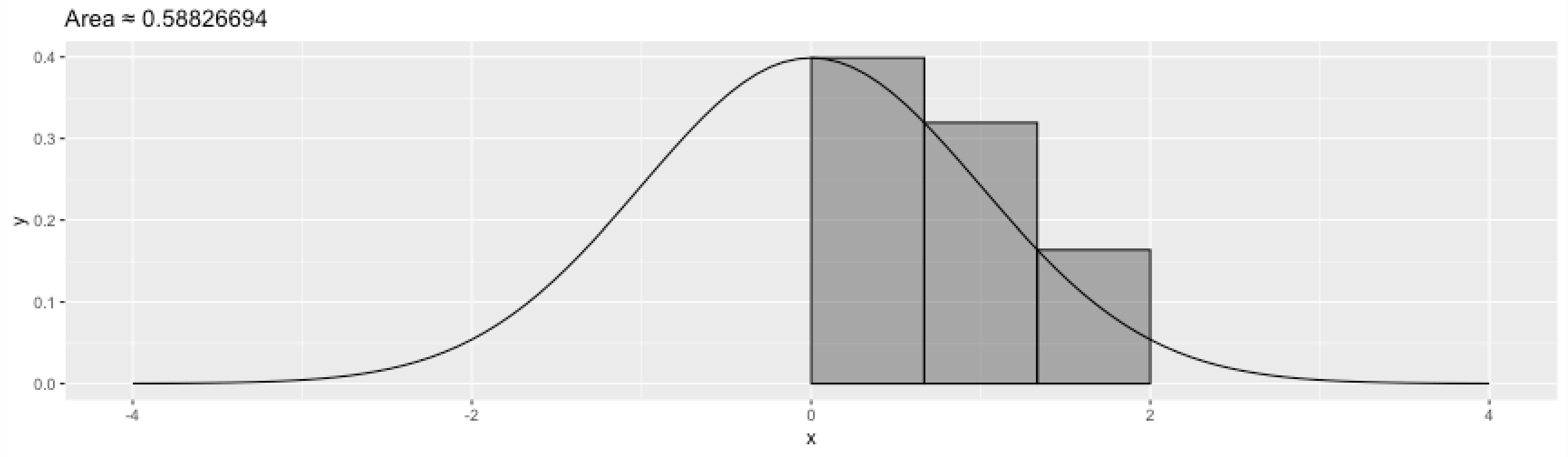
```r
f <- function(x, mean = 0, sigma = 1) {
    1 / (sigma * sqrt(2 * pi)) * exp(1)^(-1/2 * ( (x - mean) / sigma )^2)
}
```

```r
min <- 0; max <- 2
ggplot() + stat_function(fun = f) + xlim(c(-4, 4)) +
    geom_vline(xintercept = c(min, max), color = 'blue', linetype = 2) + xlab('x')
```

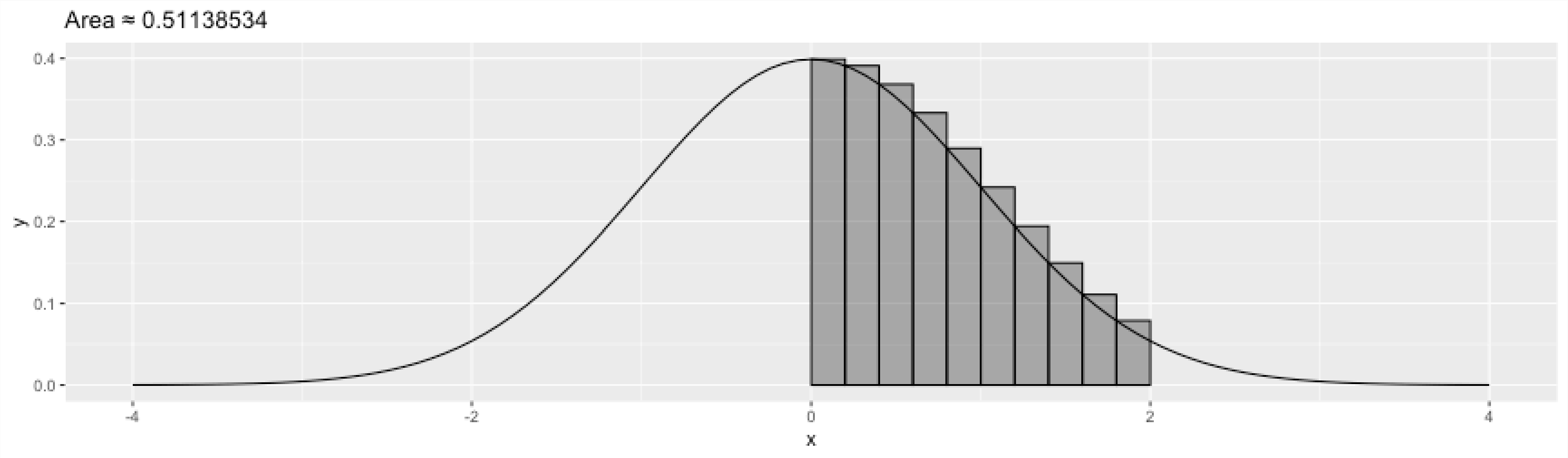


Reimann Sums

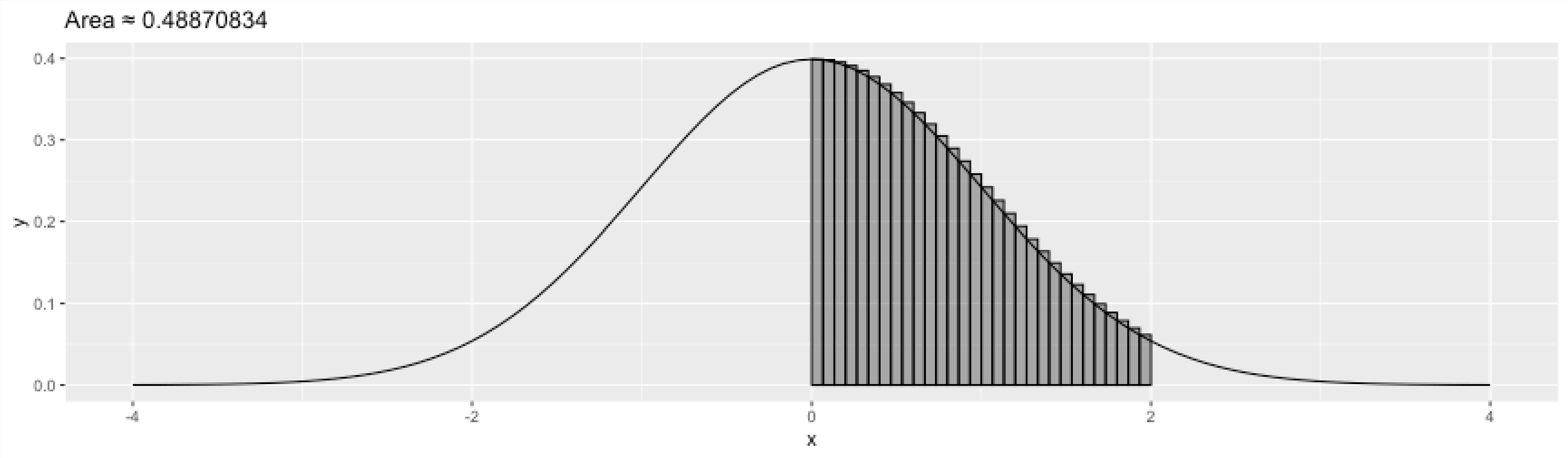
One strategy to find the area between two values is to draw a series of rectangles. Given *n* rectangles, we know that the width of each is $\frac{2-0}{n}$ and the height is $f(x)$. Here is an example with 3 rectangles.



Reimann Sums (10 rectangles)



Reimann Sums (30 rectangles)



Reimann Sums (300 rectangles)

$n \rightarrow \infty$

As *n* approaches infinity we are going to get the *exact* value for the area under the curve. This notion of letting a value get increasingly close to infinity, zero, or any other value, is called the **limit**.

The area under a function is called the integral.

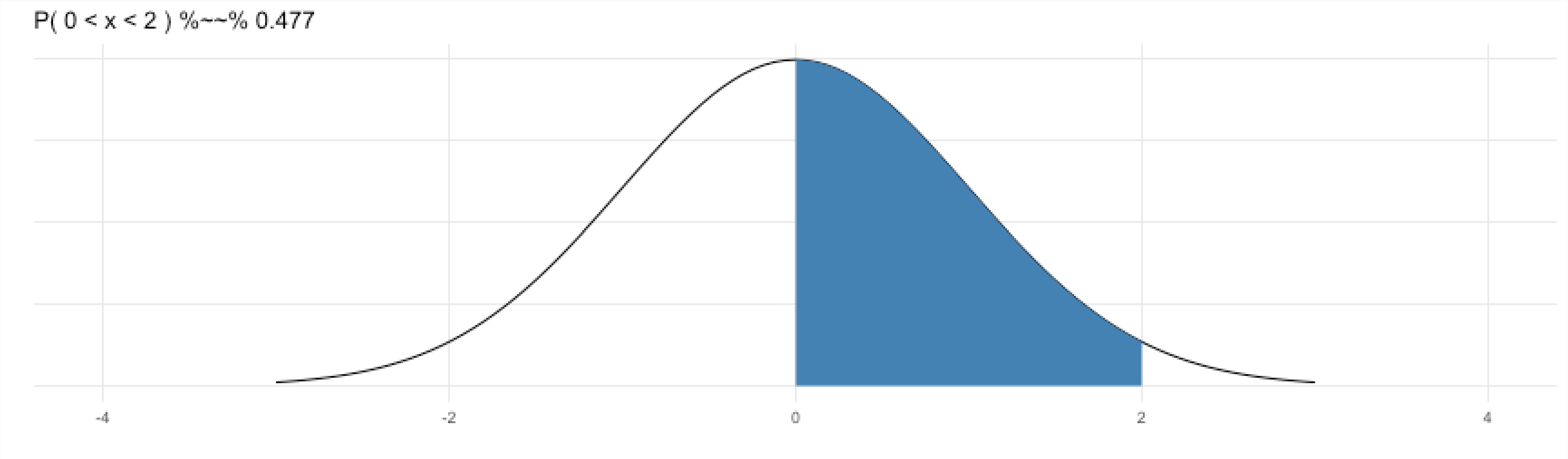
```
integrate(f, 0, 2)
```

```
## 0.4772499 with absolute error < 5.3e-15
```

```
DATA606::shiny_demo('calculus')
```

Normal Distribution

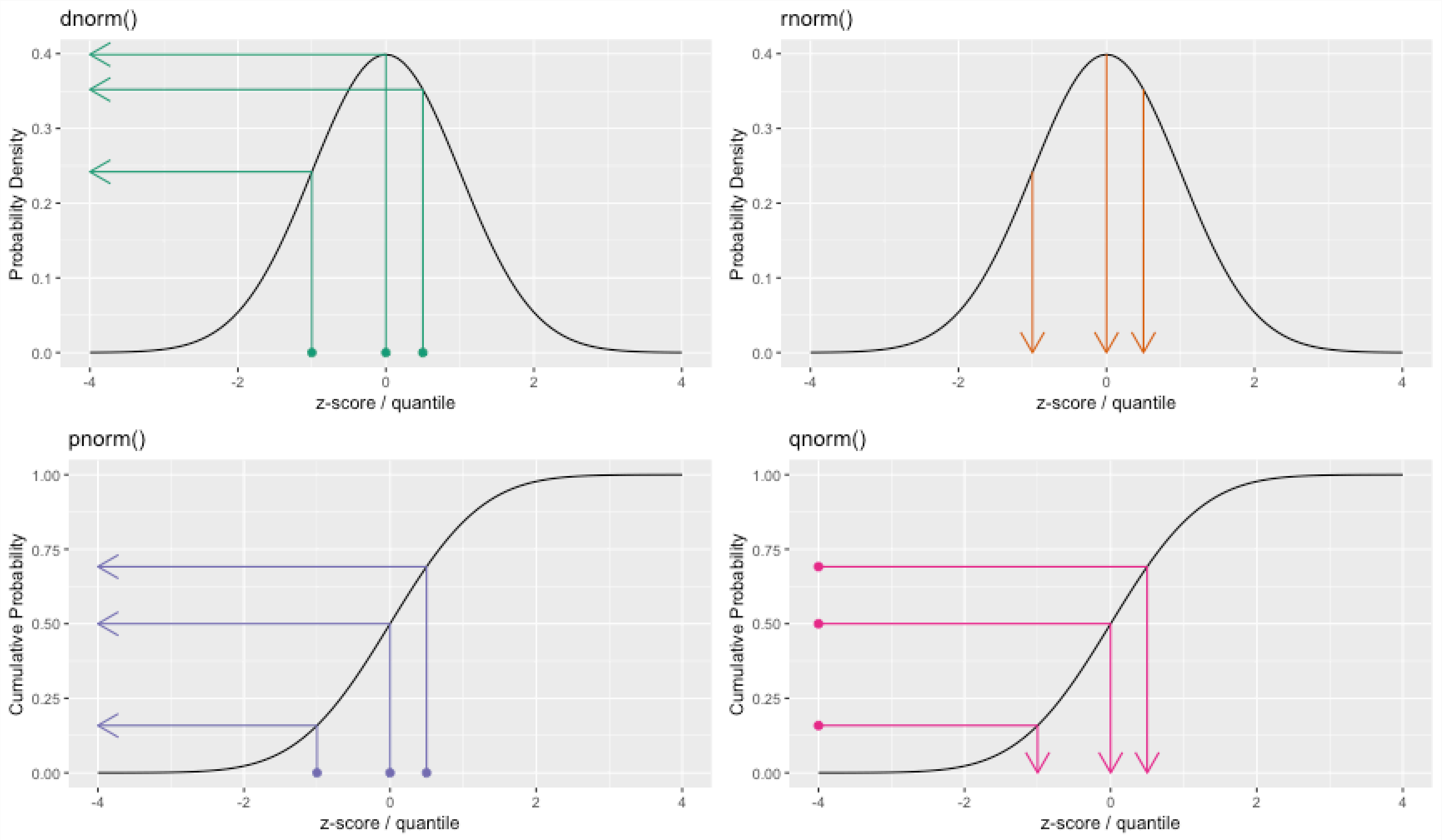
```r
normal_plot(cv = c(0, 2))
```




```r
pnorm(2) - pnorm(0)
```

```
## [1] 0.4772499
```

R's built in functions for working with distributions



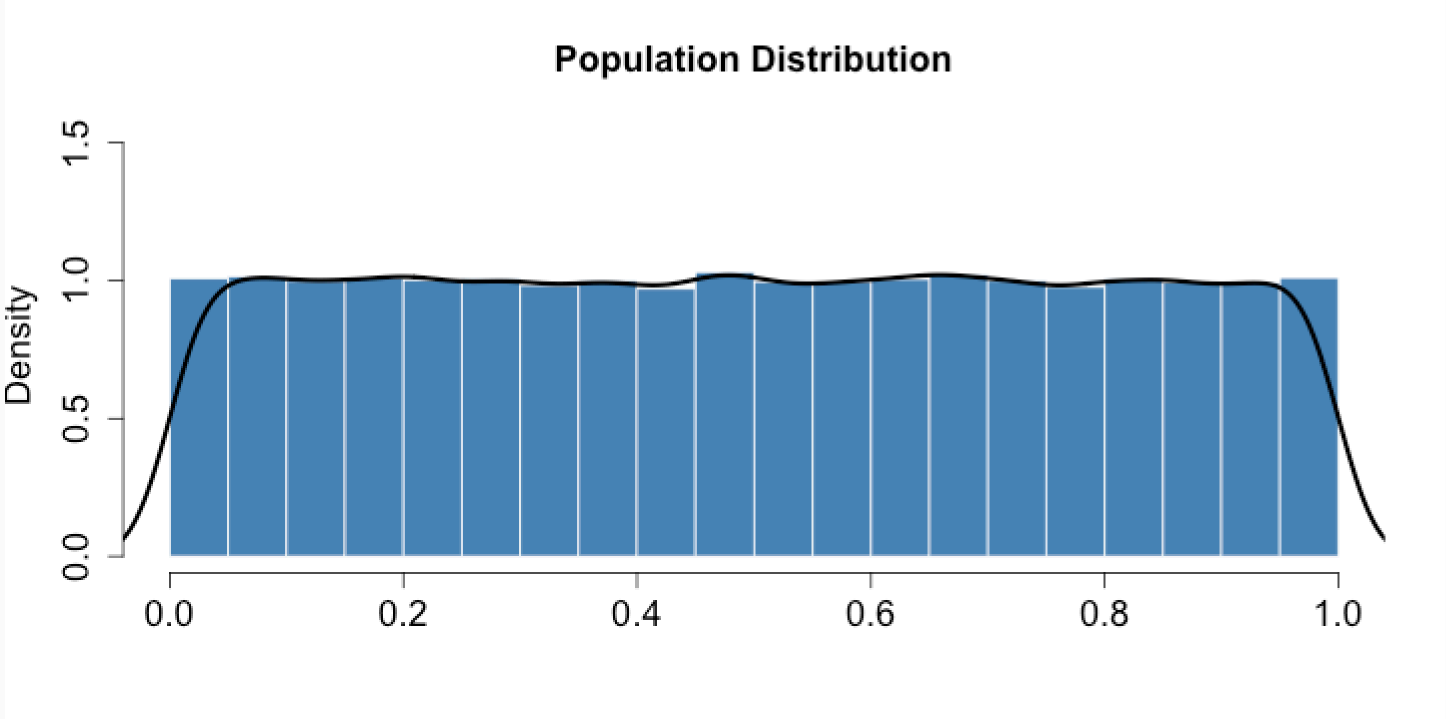
See https://github.com/jbryer/DATA606Fall2021/blob/master/R/distributions.R

Foundation for Inference

Population Distribution (Uniform)

```
n <- 1e5
pop <- runif(n, 0, 1)
mean(pop)
```

```
## [1] 0.4994449
```

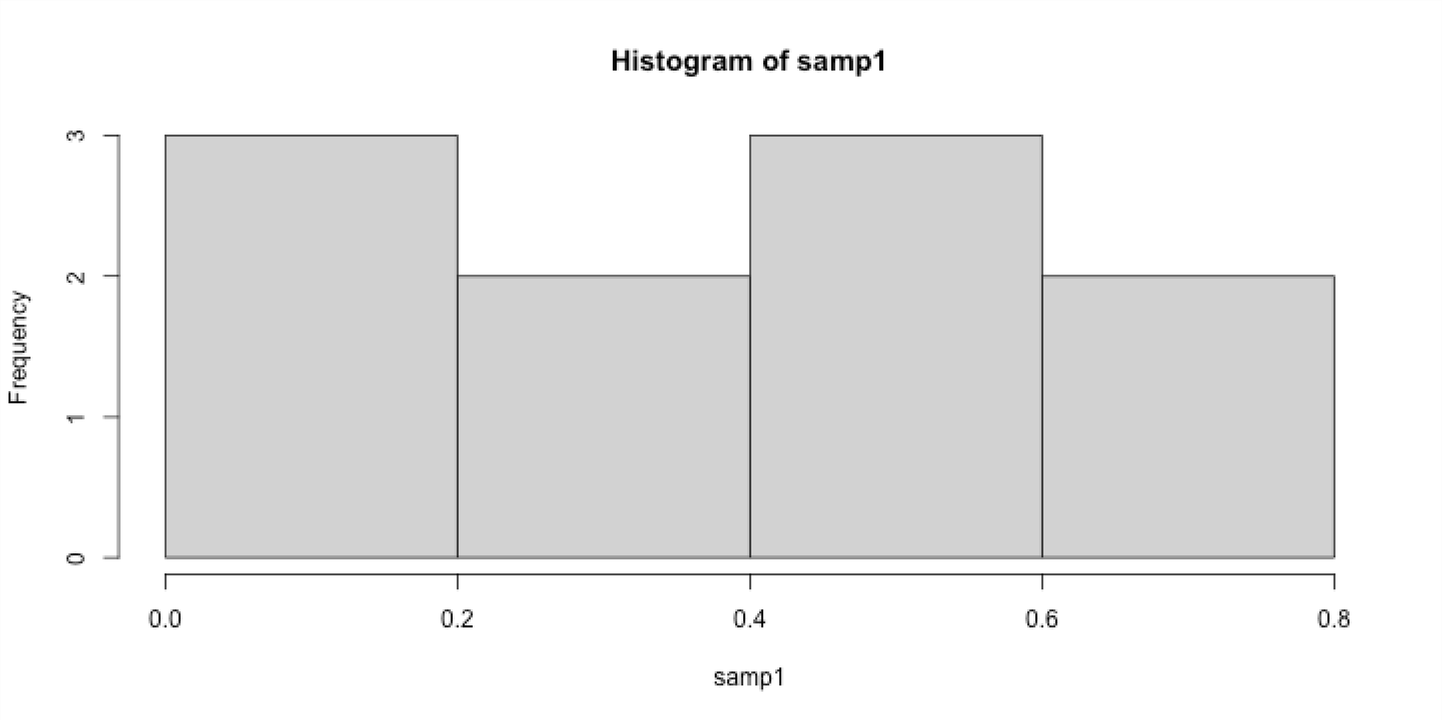


Random Sample (n=10)

```r
samp1 <- sample(pop, size=10)
mean(samp1)
```

```
## [1] 0.3934338
```

```r
hist(samp1)
```

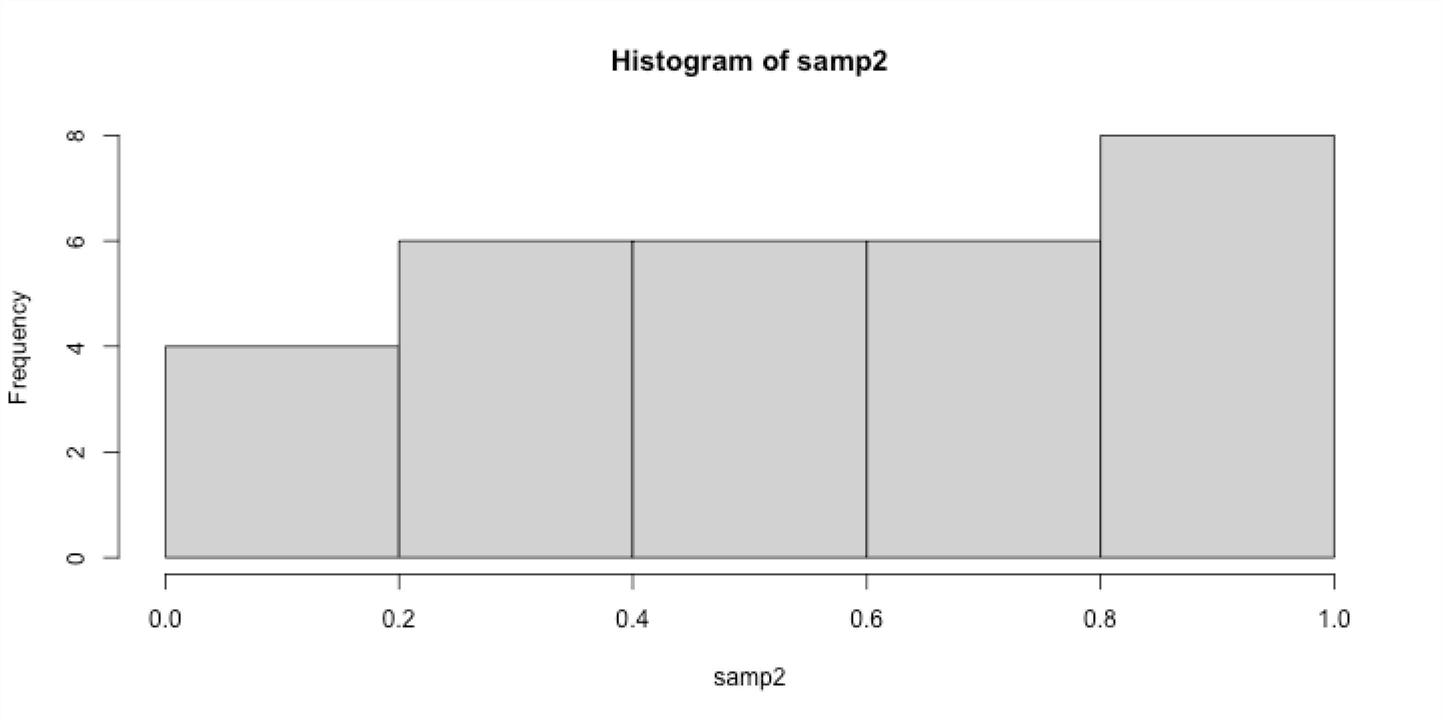


Random Sample (n=30)

```
samp2 <- sample(pop, size=30)
mean(samp2)
```

```
## [1] 0.5607104
```

```
hist(samp2)
```



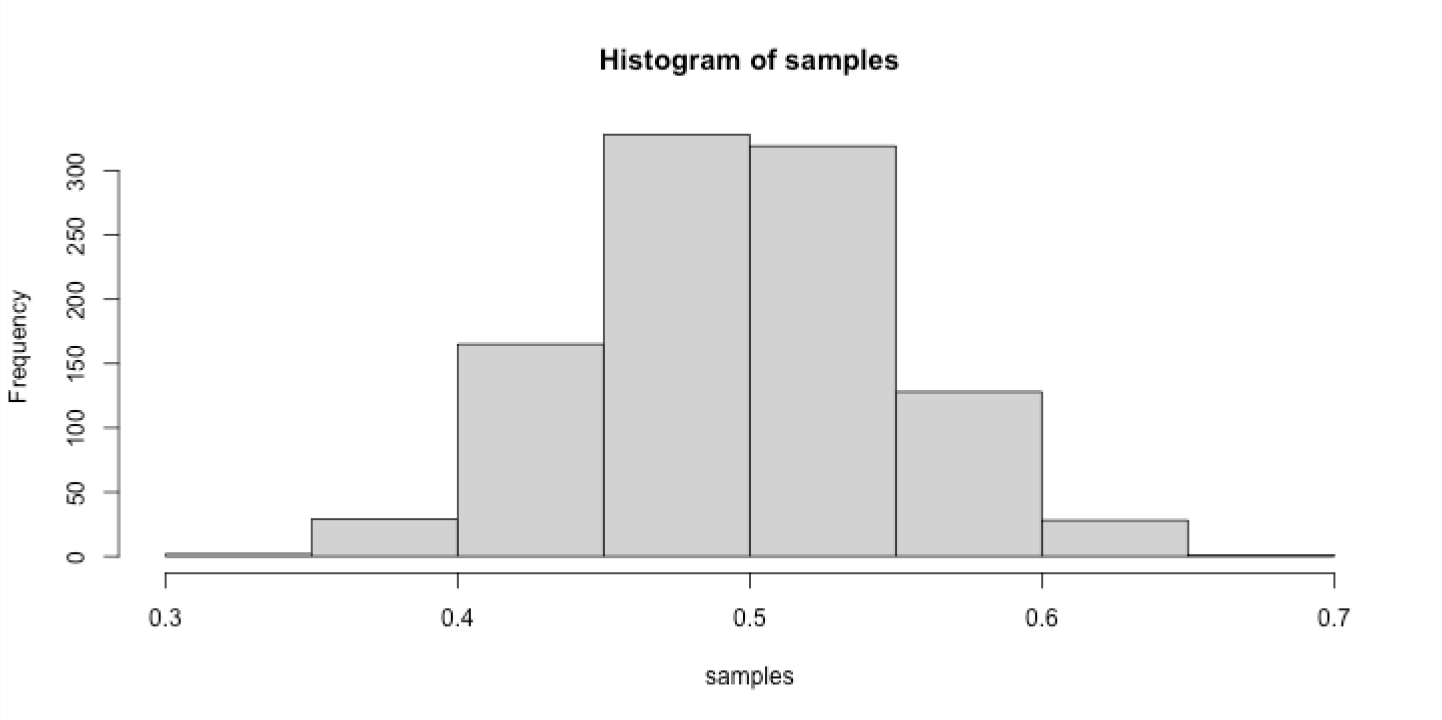
Lots of Random Samples

```r
M <- 1000
samples <- numeric(length=M)
for(i in seq_len(M)) {
    samples[i] <- mean(sample(pop, size=30))
}
head(samples, n=8)
```

```
## [1] 0.4562877 0.4175779 0.5847211 0.3979259 0.5371253 0.4668290 0.5062707
## [8] 0.4682416
```

Sampling Distribution

```
hist(samples)
```



Central Limit Theorem (CLT)

Let X_1, X_2, ..., X_n be independent, identically distributed random variables with mean μ and variance σ^2, both finite. Then for any constant z,

$$\lim_{n\to\infty} P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq z\right) = \Phi\left(z\right)$$

where Φ is the cumulative distribution function (cdf) of the standard normal distribution.

In other words...

The distribution of the sample mean is well approximated by a normal model:

$$\bar{x} \sim N\left(mean = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$$

where SE represents the **standard error**, which is defined as the standard deviation of the sampling distribution. In most cases σ is not known, so use s.

CLT Shiny App

```
library(DATA606)
shiny_demo('sampdist')
shiny_demo('CLT_mean')
```

Standard Error

```r
samp2 <- sample(pop, size=30)
mean(samp2)
```

```
## [1] 0.5499407
```

```r
(samp2.se <- sd(samp2) / sqrt(length(samp2)))
```

```
## [1] 0.04538961
```

Confidence Interval

The confidence interval is then $\mu \pm CV \times SE$ where CV is the critical value. For a 95% confidence interval, the critical value is ~1.96 since

$$\int_{-1.96}^{1.96} \frac{1}{\sigma\sqrt{2\pi}} d^{-\frac{(x-\mu)^2}{2\sigma^2}} \approx 0.95$$

```r
qnorm(0.025) # Remember we need to consider the two tails, 2.5% to the left, 2.5% to the right.
```

```
## [1] -1.959964
```

```r
(samp2.ci <- c(mean(samp2) - 1.96 * samp2.se, mean(samp2) + 1.96 * samp2.se))
```

```
## [1] 0.4609771 0.6389044
```

Confidence Intervals (cont.)

We are 95% confident that the true population mean is between 0.4609771, 0.6389044.

That is, if we were to take 100 random samples, we would expect at least 95% of those samples to have a mean within 0.4609771, 0.6389044.

```
ci <- data.frame(mean=numeric(), min=numeric(), max=numeric())
for(i in seq_len(100)) {
    samp <- sample(pop, size=30)
    se <- sd(samp) / sqrt(length(samp))
    ci[i,] <- c(mean(samp),
                mean(samp) - 1.96 * se,
                mean(samp) + 1.96 * se)
}
ci$sample <- 1:nrow(ci)
ci$sig <- ci$min < 0.5 & ci$max > 0.5
```

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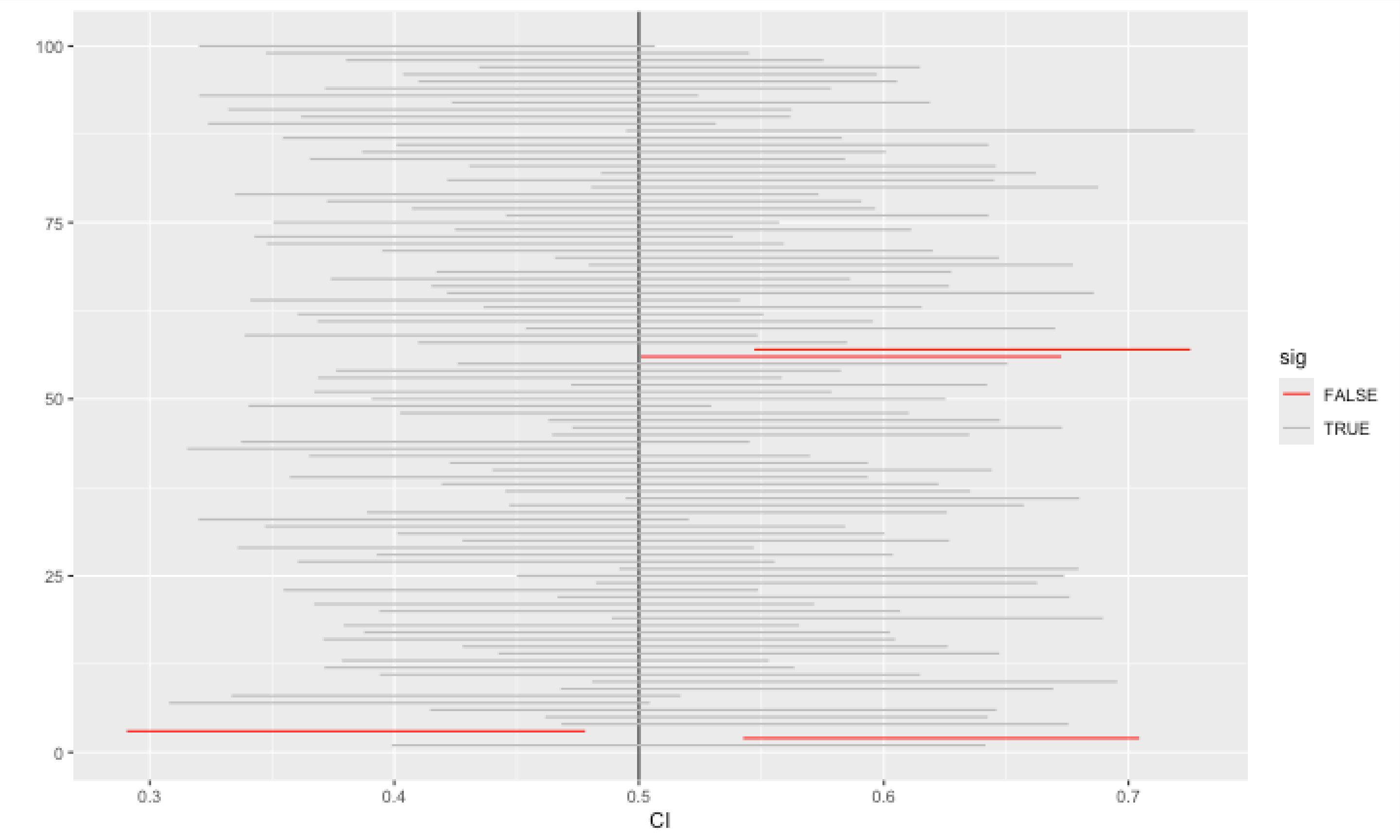
Confidence Intervals

```
ggplot(ci, aes(x=min, xend=max, y=sample, yend=sample, color=sig)) +
    geom_vline(xintercept=0.5) +
    geom_segment() + xlab('CI') + ylab('') +
    scale_color_manual(values=c('TRUE'='grey', 'FALSE'='red'))
```



One Minute Paper

1. What was the most important thing you learned during this class?
2. What important question remains unanswered for you?

https://forms.gle/N8WjTAysfKbGLptLA